Health-Related Life Cycle Risks and Public Insurance*  
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Daniel Kemptner†  
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Abstract

Based on a dynamic life cycle model, this study analyzes health-related risks of consumption and old-age poverty. The model allows for health effects on employment risks, on productivity, on longevity, the correlation between health risks, productivity and preferences, and the financial incentives of the German public insurance schemes. The estimation uses data on male employees and an extended Expectation-Maximization algorithm. Simulations suggest that health shocks induce average losses in annual consumption of about 10% and account for more than two-thirds of the cases of old-age poverty. Annuity markets that account for differences in the longevity risk by health status can effectively reduce the consumption risks, but only slightly decrease old-age poverty. A policy analysis of minimum pension benefits indicates that a means test mitigates the associated moral hazard problem substantially.

Keywords: dynamic programming, discrete choice, health, employment, early retirement, consumption, tax and transfer system.

JEL Classification: C61, I14, J22, J26

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†DIW Berlin, dkemptner@diw.de
1 Introduction

The strong association between health and socio-economic status is a robust finding and is discussed by a large body of literature (for an overview, see Cutler and Lleras-Muney, 2012; Grossman, 2006). While there is little robust evidence that income affects health in developed countries\(^1\), there is strong evidence that a substantial share of income inequality can be explained by health (Deaton, 2003). In fact, health shocks are found to be one of the main determinants of labor market participation and early retirement. Both unemployment and early retirement reduce an individual’s expected life cycle income. As pointed out by Deaton (2003), this is due to the fact that individuals cannot fully insure their earnings against health risks. For this reason, public insurance mechanisms are particularly relevant for the insurance of individuals against the risk of health-related old-age poverty.\(^2\)

Several recent studies analyze wage and employment risks within a life cycle framework (e.g. Low, 2005; Low, Meghir, and Pistaferri, 2010; Adda, Dustmann, Meghir, and Robin, 2013; Heathcote, Storesletten, and Violante, 2014). Furthermore, Low and Pistaferri (2015) investigate the insurance value of disability benefits and incentive costs. This study adds to the literature on life cycle risks by analyzing the health-related risks of consumption and old-age poverty on the basis of a dynamic life cycle model of health, employment, early retirement, and wealth accumulation. Since health shocks affect labor supply, retirement, and saving behavior, it is important to account for all three behavioral margins and their interplay with the public insurance mechanisms (i.e. unemployment insurance, social assistance, and disability pension). In particular, the model allows for health effects on employment risks, on productivity, on longevity, the correlation between health risks, productivity and preferences, and the financial incentives of the German public insurance schemes. A policy analysis investigates a budgetary neutral introduction of minimum pension benefits as insurance against old-age poverty. Following a theoretical argument of Golosov and Tsyvinski

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\(^1\)Adda, Banks, and von Gaudecker (2009) find no effect of permanent income shocks on a wide range of health measures.

\(^2\)In Germany, average pension benefits of early retirees who enter early retirement due to a work disability declined nominally from €774 to €720 between 2000 and 2012, and each year about 260,000 of these early retirees are registered newly with the statutory pension insurance (Deutsche Rentenversicherung, 2015).
that a means test may prevent false claims of benefits, I compare means-tested with non-means-tested schemes.

This study makes three main research contributions. First, I present a methodological framework that is particularly suited to quantify the health-related risks of consumption and old-age poverty. It not only accounts for all relevant behavioral margins and the public insurance schemes, but also allows for the correlation of unobserved heterogeneity in the health risks, productivity and preferences through a joint estimation of the behavioral model, the wage equation, and the health transitions (for a discussion of these mechanisms, also see Capatina, 2015; French and Jones, 2017). This is implemented using an extension of the Expectation-Maximization (EM) algorithm (as proposed by Arcidiacono and Jones, 2003). Second, I apply this framework to quantify losses in average annual consumption and the share of old-age poverty that can be attributed to health shocks. This is done by comparing simulated outcomes for a representative synthetic sample with the respective outcomes of a simulated benchmark scenario where no health shocks occur. Third, the results of this analysis motivate the simulation of minimum pension benefits as insurance against old-age poverty. While this raises the level of public insurance for health-related risks, it also raises a concern about an increase in abuse of the disability pension. However, the simulations suggest that a means test mitigates the moral hazard problem substantially. This finding supports the theoretical argument of Golosov and Tsyvinski (2006).

The life cycle model is based on a dynamic programming discrete choice (DPDC) framework (similar to e.g. Rust and Phelan, 1997; French, 2005; van der Klaauw and Wolpin, 2008; French and Jones, 2011; Haan and Prowse, 2014) that accounts for dynamic incentives induced by public insurance schemes and path dependencies of health and employment. The model serves two main purposes: (1) It allows deriving life cycle implications of health shocks and reform scenarios under current institutions and for individuals that have been recently observed in the labor force (no historical institutions and data); (2) behavioral responses can be taken into account when simulating counterfactual scenarios. The model is estimated using data on male employees from the German Socio-Economic Panel (SOEP) and by applying an extension of the EM algorithm. The algorithm has been proposed by Arcidiacono and Jones (2003) for the estimation of finite mixture models with time-constant unobserved heterogeneity and
allows for a sequential estimation of the parameters, see Arcidiacono (2004, 2007) for other applications. Following Arcidiacono and Jones (2003), the EM algorithm is used to obtain good starting values for a subsequent efficient full information maximum likelihood (FIML) procedure.

Based on the parameter estimates, I use the life cycle model to simulate scenarios where health shocks either occur according to the estimated health process, or do not occur at all, or occur at a certain age and are persistent (no transition back to good health). A comparison of these simulations suggests that health shocks induce average losses in annual consumption of about 10% and account for more than two-thirds of the cases of old-age poverty. Both average losses in consumption and changes in the risk of old-age poverty depend substantially on initial endowments. Average losses in consumption are severe if a persistent health shock occurs at an early stage in the life cycle. The simulation of the introduction of different levels of minimum pension benefits indicates only a small decrease in average lifetime employment and no decrease in average retirement age if the minimum benefits are means-tested. For a non-means-tested scheme, the simulations suggest a severe moral hazard problem for low productivity types. A robustness check suggests that annuity markets that account for differences in the longevity risk by health status can effectively reduce the consumption risks, but only slightly decrease health-related old-age poverty.

The paper is structured as follows. I begin with an outline of the life cycle model. Then, I proceed with a description of the data before discussing the estimation approach, the parameter estimates and model fit. The following section presents the policy analysis on the health-related consumption and poverty risks and the counterfactual reforms. Subsequently, I investigate the robustness of the results. A final section sums up the main findings of the analysis.

2 Model and specification

2.1 Main features

Framework: Individuals maximize their expected lifetime utility by making choices about employment, early retirement and saving behavior in each period of time (annual data). They face uncertainty in terms of health, labor market and longevity risks.
The set of possible choices is restricted by eligibility requirements for early retirement and by job offer and separation rates. Individuals have rational expectations and face a dynamic programming problem with a finite horizon. Unlike studies that apply a two-step estimation approach (e.g. French, 2005; French and Jones, 2011), the model accounts for the correlation of unobserved heterogeneity in health risks, productivity, and individual preferences. This is implemented by allowing for a finite number of unobserved types (Heckman and Singer, 1984). In more detail, there are three unobserved types that differ in terms of their preferences, health risks and labor productivity (high, medium, low). Each of these types comprises a fixed proportion of the individuals in the population. The correlation between preferences and productivity captures selection into the labor market.

**Health:** Health is modeled as a binary second order autoregressive process that depends also on years of education, and age. Since it is unobserved whether a health shock leads to a work disability, the model assumes that bad health affects employment and early retirement through the labor market risks (captured by job offer and separation rates), labor productivity, longevity, the eligibility requirements for early retirement (disability pension), and the financial incentives (disability pension is more generous than a regular pension). Unemployed individuals who are in bad health status may have a strong incentive to opt for early retirement because the forward-looking agents take into account their low job offer rates (low opportunity costs).

Unlike Grossman (1972), the model does not allow for health investments that may reduce the impact of a health shock. This is in line with e.g. Rust and Phelan (1997), French (2005), De Nardi, French, and Jones (2009), French and Jones (2011), and Low and Pistaferri (2015), but differs from the approach taken by other studies that explicitly account for health investments (e.g. Hall and Jones, 2007; Blau and Gilleskie, 2008; Hugonnier, Pelgrin, and St-Amour, 2013). The abstraction reduces the complexity of the model and makes it numerically feasible to account for other important sources of heterogeneity like the unobserved productivity types that exhibit heterogeneous health risks and preferences. However, I include net wealth in the health transition equation and show that the means available for out-of-pocket health expenditures have no significant effect. This is likely due to the high level of general health insurance being provided in Germany.
Budget: Wages are estimated within the model (as a function of education, work experience, and health). Pension benefits are a deterministic function of retirement age, health at retirement, work experience, and past wages that are reconstructed using the wage equation. This induces dynamic incentives that are captured by the DPDC framework. When making employment choices, individuals take into account both the effect of human capital accumulation on wages (Eckstein and Wolpin, 1989) and the effect on future pension claims. Net income is computed by applying the rules and regulations of the German tax and transfer system and statutory pension insurance scheme, where unemployment insurance, social assistance and disability pension constitute a partial insurance against work disability.

The model includes the following state variables: age, net wealth, work experience, years of education, health status, lagged health status, employment restriction, and previous period’s choices. The choice problem consists of two parts: i) an optimal stopping problem with respect to retirement and ii) employment and saving choices.

2.2 Objective function

I set up a DPDC model of individuals’ employment, early retirement and saving choices. Individuals are finitely lived and die no later than period T, which is set to be age 100. Discrete time is indexed by t (individual’s age), and there is a number of N individuals, indexed by n. Each individual n receives a utility flow $U(s_{nt}, d_{nt})$ in period t where $s_{nt}$ is the vector of state variables, and $d_{nt}$ indicates the individual’s choice. Every period t, an individual n observes the state variables $s_{nt}$ and selects the choice $d_{nt}$ from the choice set $D(s_{nt})$ that maximizes expected lifetime utility:

$$
E \left[ \sum_{j=0}^{T-t} p(t+j|t, s_{nt}) \beta^j U(s_{nt+j}, d_{nt+j}) \right]
$$

where $\beta$ is the time discount factor (set to be 0.97) and $p(t+j|t, s_{nt})$ is the conditional survival probability for period $t+j$ given survival until period t. The model accounts for differential survival probabilities by health status and the level of education (≥ or < 12 years). Health effects on survival probabilities are relevant for this analysis to the degree that objective differences between the healthy and the unhealthy translate into different subjective expectations that affect behavior. Evidence by e.g. Hamermesh (1985) and
Hurd and McGarry (1995, 2002) suggest that there is indeed some consistency between objective mortality and subjective mortality expectations. Details on the estimation of the heterogeneous survival probabilities are provided in Appendix J.

The choice set is restricted by the eligibility requirements for early retirement (see Appendix A.2 for details) and by job offer and separation rates that are estimated within the model. Before retirement, individuals choose between working, not working and early retirement. Furthermore, they make saving choices that are approximated by a number of grid points. Individuals retire no later than the regular pension age of 65. After retirement, they make no more choices and buy an actuarially fair life annuity with their accumulated net wealth. The fair life annuity is calculated based on average age-specific survival rates and by assuming a real interest rate of 2%. In a robustness check, I also consider a specification where the annuity market accounts for differences in the longevity risk by health status and education.

2.3 Utility function

Individuals have preferences about consumption and leisure time that are represented by the following time separable random utility model, where the random component $\epsilon_{nt}(d_{nt})$ is assumed to be type 1 extreme value distributed:

$$U(s_{nt}, d_{nt}) = \alpha_n c(s_{nt}, d_{nt}) \frac{(1-\rho_m) - 1}{(1 - \rho_m)} + \epsilon_{nt}(d_{nt})$$

$$\alpha_n = a_1 + a_{2m} \text{work}(d_{nt})$$

$c(s_{nt}, d_{nt})$ is the level of consumption associated with state $s_{nt}$ and choice $d_{nt}$. $\text{work}(d_{nt}) \in \{0, 1\}$ indicates employment. The parameters $a_{2m}$ (disutility of work) and $\rho_m$ (coefficient of relative risk aversion) capture preference heterogeneity by unobserved types $m \in \{1, ..., M\}$. $\alpha_n$ is a consumption weight that depends on a constant (scaling factor) and the parameter $a_{2m}$. This utility function accounts for non-separability between consumption and disutility of work, but assumes additive separability between its ob-

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3 Taking into account the empirical saving pattern, the grid differs by employment status and captures the fact that some unemployed individuals are dissaving. I define 9 grid points for the employed and 5 grid points for the unemployed. The results are insensitive to an increase in the number of grid points and to the location of these points. In the simulations, the approximation error cancels out over the individuals’ life cycles.
served component and its unobserved random component. Health does not enter the utility function, but affects employment outcomes through the labor market risks, labor productivity, longevity, the eligibility requirements for early retirement (disability pension), and the financial incentives of the early retirement scheme. Conditional on health affecting job offers and separations, the estimate for a health effect on the disutility of work is small and insignificant. In a robustness check, I also consider a specification where offer rates are set to 1 and separation rates are set to 0 and where health, education, and age are interacted with the disutility of work inside the CRRA preferences (similar to e.g. French, 2005). The vector $\theta_U = (a_1, a_{21}, \ldots, a_{2M}, \rho_1, \ldots, \rho_M)$ contains the parameters of the utility function.

2.4 Job offer and separation rates

An individual’s choice of employment is restricted by job offer and separation rates that are estimated within the model. The offer rates capture persistence of the unemployment status. Individuals who have been unemployed in the previous period may choose employment only if they receive a job offer in the current period. Analogously, individuals who have been employed in the previous period may choose employment only if they do not face a job separation in the current period. The rates are estimated differentially by education ($\geq$ or $<$ 12 years), health status, and age:

$$\Pr(\text{offer}_{nt} = 1) = \Lambda(\phi_1 + \phi_2 \text{educ}_{n}^{12+} + \phi_3 \text{health}_{nt} + \phi_4 \text{age}_{nt}^{50+} + \phi_5 \text{age}_{nt}^{55+} + \phi_6 \text{age}_{nt}^{60+})$$

$$\Pr(\text{sep}_{nt} = 1) = \Lambda(\phi_7 + \phi_8 \text{educ}_{n}^{12+} + \phi_9 \text{health}_{nt} + \phi_{10} \text{age}_{nt}^{50+} + \phi_{11} \text{age}_{nt}^{55+} + \phi_{12} \text{age}_{nt}^{60+})$$

where $\Lambda(\cdot)$ is the logistic distribution function. The age dummies indicate an age greater or equal than 50, 55, and 60 years, respectively. Individuals losing their previous job or lacking a current job offer can only choose between unemployment and early retirement (if eligible). It follows from the persistence of the unemployment status - taken into account by forward looking individuals - that unemployed individuals may have a strong incentive to opt for early retirement (in particular when they are in bad health). The parameters are contained by the vector $\phi = (\phi_1, \phi_2, \phi_3, \phi_4, \phi_5, \phi_6, \phi_7, \phi_8, \phi_9, \phi_{10}, \phi_{11}, \phi_{12})$. These parameters are identified separately from those characterizing the disutility of work through the observed transitions in and out of unemployment.
2.5 Health transitions

Health is modeled as a binary second order autoregressive process that depends also on years of education and health. The lag structure takes into account path dependencies and captures the fact that transition probabilities from bad to good health status are smaller if health shocks have lasted for a longer period of time. Education captures differences in the health risks by socio-economic status.\(^4\) The probability of good health is given by

\[
\Pr(\text{health}_{nt} = 1) = \Lambda(\psi_{1m} + \psi_2\text{health}_{n-1} + \psi_3\text{health}_{n-2} + \psi_4\text{educ}_n + \psi_5\text{wealth}_n + \psi_6\text{age}^{30+}_n + \psi_7\text{age}^{40+}_n + \psi_8\text{age}^{50+}_n + \psi_9\text{age}^{60+}_n)
\]

where \(\Lambda(\cdot)\) is the logistic distribution function, \(\text{health}_{nt}\) is a binary variable that indicates good health status, \(\text{educ}_n\) is years of education, and a set of age dummies indicate an age greater or equal than 30, 40, 50, and 60 years, respectively. Heterogeneity in health risks by unobserved types is captured by \(\psi_{1m}\).

As pointed out in section 2.1, the health equation does not include health investments because medical expenditures are covered by general health insurance.\(^5\) However, I allow health transitions to depend on net wealth to show that the effect is very small and that the means available for health investments through out-of-pocket expenditures are not affecting the health outcomes. The parameters of the health transitions are contained by the vector \(\theta_h = (\psi_{11}, \ldots, \psi_{1M}, \psi_2, \psi_3, \psi_4, \psi_5, \psi_6, \psi_7, \psi_8, \psi_9)\).

\(^4\)Quasi-experimental evidence suggests that education even exerts a causal effect on health and health behavior (for Germany, see e.g. Kemptner, Juerges, and Reinhold, 2011).

\(^5\)According to statistics of the federal statistical office (https://www-genesis.destatis.de, accessed on 26.7.2018), the private health expenditures in Germany account for about 15% of overall medical expenditures. However, for employees the public health insurance covers all medically indicated health expenditures (exceptions: a very low fix fee of 10 Euro that had to be paid until 2012 once per quarter of a year when seeing a doctor, a maximum 10 Euro deductible for prescription drugs, 10 Euro per day in hospital for the first 28 days). Hence, most of the out-of-pocket expenditures are not health investments, but rather consumption in the sense that they make the medical treatment a bit more pleasant (e.g. some patients pay extra for a single room when being hospitalized).
2.6 Gross wage

Gross wages are assumed to follow a log-normal distribution. The logarithm of gross wages is modeled as

$$\log(\text{wage}_{nt}) = \delta_{1m} + \delta_2 \text{educ}_n + (\delta_3 \text{exper}_{nt} + \delta_4 \text{exper}^2_{nt}) \times (\text{educ}_n < 12) + (\delta_5 \text{exper}_{nt} + \delta_6 \text{exper}^2_{nt}) \times (\text{educ}_n \geq 12) + \delta_7 \text{health}_{nt} + \mu_{nt}$$

where $\text{educ}_n$ is years of education, $\text{exper}_{nt}$ is years of work experience, and $\mu_{nt}$ is i.i.d. $\mathcal{N}(0, \sigma_\mu)$. In line with e.g. Low, Meghir, and Pistaferri (2010), transitory variation in $\log(\text{wages})$ is interpreted as measurement error. This simplifies the estimation of the model substantially and the assumption is of little importance for the simulation of life cycle outcomes because transitory wage shocks even out over the life cycle. $\delta_{1m}$ captures heterogeneity in labor productivity by unobserved types.

It is due to the DPDC framework that individuals take into account the human capital accumulation process when making their employment choice (Eckstein and Wolpin, 1989). Hence, work experience is an endogenous variable in the model. The interaction terms between work experience and education account for heterogeneous returns to work experience for the high and the low educated (as reflected by diverging wage profiles in the sample data). The health effect captures changes in individuals’ productivity that are induced by a health shock. The correlation between type-specific preferences and the unobserved component, $\delta_{1m}$, in the wage equation accounts for selection into the labor market. When computing gross labor earnings, I assume that individuals work the median number of hours, which is 40 in the sample. The vector $\theta_w = (\delta_{11}, \ldots, \delta_{1M}, \delta_2, \delta_3, \delta_4, \delta_5, \delta_6, \delta_7, \sigma_\mu)$ contains the parameters of the wage equation.
2.7 Budget constraint

Individuals face a budget constraint when making their saving/consumption choice that is defined by the following equations:

\[
c(s_{nt}, d_{nt}) = \text{net income}_{nt}(s_{nt}, d_{nt}) - \text{saving}(d_{nt})
\]

\[
\text{net income}_{nt}^{\text{pre ret.}}(s_{nt}, d_{nt}) = \text{wage}(s_{nt}, d_{nt}) + \text{transfers}(s_{nt}, d_{nt})
\]
\[
- \text{taxes}(s_{nt}, d_{nt}) - \text{contributions}(s_{nt}, d_{nt})
\]

\[
\text{net income}_{nt}^{\text{post ret.}}(s_{nt}, d_{nt}) = \text{pension}(s_{nt}, d_{nt}) + \text{annuity}(s_{nt}, d_{nt}) + \text{transfers}(s_{nt}, d_{nt})
\]
\[
- \text{taxes}(s_{nt}, d_{nt}) - \text{contributions}(s_{nt}, d_{nt})
\]

\[
\text{transfers}(s_{nt}, d_{nt}) = \text{unempl. ins. benefits}(s_{nt}, d_{nt}) + \text{social assistance benefits}(s_{nt}, d_{nt})
\]

\[
\text{wealth}_{nt+1} = (1 + r_t)(\text{wealth}_{nt} + \text{saving}(d_{nt}))
\]

\[
\text{wealth}_{nt} \geq 0
\]

where \(c(s_{nt}, d_{nt})\) is the level of consumption associated with state \(s_{nt}\) and choice \(d_{nt}\).

Net income for the pre- and post-retirement period is computed by applying the rules and regulations of the German tax and transfer system and of the statutory pension insurance scheme. A detailed outline of the institutions is provided in Appendix A. I assume that individuals do not expect future changes in the institutional framework.

\(\text{wealth}_{nt}\) is period \(t\)'s net wealth, \(r_t\) is the real interest rate that is set to be 0.02, and \(\text{saving}(d_{nt})\) is the amount of saving associated with choice \(d_{nt}\).

When working, individuals receive a gross wage that is determined by the wage process being outlined in the previous subsection. After retirement, they receive pension benefits and an annuity income. Pension claims are a deterministic function of retirement age, health at retirement (disability pension), work experience, and past wages that are reconstructed using the wage equation. For this purpose, I compute the individuals pension points and derive the pension benefits based on the retirement age and type of pension by following the institutional rules that are outlined in Appendix A.2. The wage history can be approximated based on the state variables of the model because the only transitory component in the wage equation (health effect) is

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6Since only minor changes occurred to the institutional framework between 2005 and 2012 (survey years of estimation sample), I apply the rules and regulations from the year 2005 to all survey years. This saves computational time because it is not necessary to estimate a different value function for each survey year.
estimated to be very small and can, therefore, be neglected. The statutory pension age is 65 years. Individuals can opt for early retirement from age 60 onwards when being in good health. Individuals who are in bad health can claim a disability pension at any age. For the disability pension, individuals receive benefits with a reduced early retirement penalty and the claims are computed as if the individual had kept working and contributing to the pension system until age 60. The annuity income follows from the assumption that individuals buy a fair life annuity at retirement with their accumulated net wealth.

Transfers comprise unemployment insurance benefits and social assistance benefits. The model assumes that all individuals are entitled to one year of unemployment insurance benefits that make up 60% of an individual’s last net earnings. Unlike the unemployment insurance benefits, social assistance benefits are paid indefinitely and are means-tested. They ensure a minimum monthly consumption level of €695 and annual benefits are equal to \( \max\{695 \times 12 - \max\{\text{wealth}_{nt} - 10,000; 0\} - \text{net income}^*; 0\} \), where \( \text{net income}^* \) is net income before social assistance benefits. The exemption level of €10,000 is assumed because the actual rules are very complicated and enforcement of these rules is unobserved.

At last, individuals have to pay a progressive income tax on both their gross wages and capital income. Moreover, they pay mandatory social security contributions for health, pension, and unemployment insurance. After retirement, individuals only pay contributions for health insurance.

### 2.8 Value function

Individuals’ beliefs about future states are captured by a Markov transition function \( q(s_{nt+1}|s_{nt}, d_{nt}) \) that indicates the transition probabilities. In particular, \( q(s_{nt+1}|s_{nt}, d_{nt}) \) captures expectations about transitions of the health status and the probabilities of receiving a job offer (if unemployed) or a job separation (if employed). For state variables like net wealth or work experience that evolve deterministically for given choices, the probability of the determined state is one while it is zero for all other possible states. Since there is a discrete set of possible future states, \( q(s_{nt+1}|s_{nt}, d_{nt}) \) is a probability mass function. By Bellman’s principle of optimality, the value function \( V_t(s_{nt}) \) can be
represented recursively as

\[
V_t(s_n, d_n) = \max_{d_n \in D(s_n, d_n)} U(s_n, d_n) + 
p(t+1|t, s_n)\beta \int \sum_{s_{n+1}} V_{t+1}(s_{n+1}) q(s_{n+1}|s_n, d_n) g(\epsilon_{n+1})
\]

where \(g(\cdot)\) is the probability density function of the unobserved random components of the utility function. \(D(s_n)\) is the choice set available to individual \(n\) in period \(t\).

### 2.9 Unobserved heterogeneity

Following Heckman and Singer (1984), unobserved heterogeneity is accounted for seminonparametrically by allowing for a finite number of unobserved types \(m \in \{1, \ldots, M\}\). Each type comprises a fixed proportion of the individuals in the population. The probability that individual \(n\) is of type \(m\) is given by \(\pi_{nm}(\gamma)\), where \(\pi_{nm}(\gamma)\) is \(\gamma^l_m\) for individuals with low education (\(< 12\) years) and \(\gamma^h_m\) for individuals with high education (\(\geq 12\) years). \(\gamma^l_M\) and \(\gamma^h_M\) are normalized to 1\(−\sum_{m=1}^{M-1} \gamma^l_m\) and 1\(−\sum_{m=1}^{M-1} \gamma^h_m\), respectively. Conditional on education, non-randomness of initial conditions is a minor concern in this model because it captures full life cycles from the moment when individuals enter the labor force. At this point in the life cycle, heterogeneity in the state variables is small. In the simulations individuals are assumed to differ only with respect to their unobserved types and years of education when entering the labor force.

### 2.10 Choice probabilities and log-likelihood

Given the finite horizon of the individual’s optimization problem, it can be solved recursively. The expected value function, \(v_t(s_n, d_n)\), is given by the flow utility for period \(T\) and has a closed form solution for periods \(T-1\) to 1 (Rust, 1987):

\[
v_T(s_nT, d_nT) = u(s_nT, d_nT)
\]

\[
v_t(s_n, d_n) = u(s_n, d_n) + p(t+1|t, s_n)\beta \times \sum_{s_{n+1}} \sum_{d_{n+1} \in D(s_{n+1})} \exp(v_{t+1}(s_{n+1}, d_{n+1})) q(s_{n+1}|s_n, d_n)
\]

\[
\forall t = 1, \ldots, T - 1
\]
The computation of the expected value functions from age 65 onwards is comparatively simple because individuals only make choices before retirement. Rust (1987) shows that under the assumptions of additive separability and conditional independence, the conditional choice probabilities have a closed form solution (mixed logit probabilities):

$$\Pr(d_{nt}|s_{nt}) = \frac{\exp(v_t(s_{nt}, d_{nt}))}{\sum_{j \in D(s_{nt})} \exp(v_t(s_{nt}, j))}$$

When computing choice probabilities, I take into account that the choice of employment is restricted by the job offers and separations. The expected value functions are computed for a discretized state space in order to save computational time (Keane and Wolpin, 1994). As a consequence, interpolation methods must be used to approximate the functions at the observed values of the state variables.\footnote{I interpolate the expected value functions for net wealth (6 grid points), work experience (6 grid points), and years of education (4 grid points). The results are insensitive to an increase in the number of grid points or the choice of interpolation function. In total, the value function is computed for 1152 \times M grid points for each of the choices. Aside the points for net wealth, work experience, and years of education, the grid comprises points for the binary state variables health status, lagged health status, lagged employment status and the unobserved types.}

The log-likelihood function of the sample is given by

$$\sum_{n=1}^{N} \log \left\{ \sum_{m=1}^{M} \pi_{nm}(\gamma) \prod_{t=1}^{T} \mathbb{L}_m(d_{nt}|\theta_U, \phi, \theta_h, \theta_w, s_{nt}) \mathbb{L}_m(health_{nt}|\theta_h, s_{nt}, s_{nt-1}) \mathbb{L}_m(wage_{nt}|\theta_w, s_{nt}) \right\}$$

where $\mathbb{L}_m(d_{nt}|\theta_U, \phi, \theta_h, \theta_w, s_{nt})$ is the likelihood contribution of the observed choice $d_{nt}$ of individual $n$ in period $t$, if $n$ is of type $m$. The likelihood contributions of the health status and wages are given by $\mathbb{L}_m(health_{nt}|\theta_h, s_{nt}, s_{nt-1})$ and $\mathbb{L}_m(wage_{nt}|\theta_w, s_{nt})$, respectively. This log-likelihood function captures intertemporal correlations between the observed choices, the health status, and the gross wages. In particular, the function accounts for the joint distribution of these variables because the health status and the other determinants of the gross wages enter the vector of state variables $s_{nt}$. Therefore, the likelihood contributions of the choices depend on observed health status and on the gross wages (corrected for measurement error) that follow from the observed values of the wage determinants together with the parameters $\theta_w$ of the wage equation.
3 Data and descriptive statistics

My analysis is based on data from the German Socio-Economic Panel (SOEP), which collects annual information at both the household and individual levels (Wagner, Frick, and Schupp, 2007). I construct an unbalanced panel covering the years 2003 through 2012. The sample is restricted to West German males aged 20 to 64 years. Self-employed, civil servants, and people in institutions are also excluded from the sample. The final sample consists of 3,078 independent and, in total, of 15,262 observations (on average: about 5 observations per individual). There are 159 individuals who opt for early retirement during the observation period and two-thirds of these individuals are in bad health when they make the early retirement choice. The consumer price index is used to adjust nominal variables to 2005 prices. Table 1 presents some descriptive statistics on the variables that are used in the analysis. These variables are discussed in more detail in the following paragraphs. Table 5 in Appendix C compares some key statistics of the sample with the respective statistics based on external administrative data sources. This comparison supports that the SOEP sample is representative for male employees in West Germany.

Table 1: Descriptive statistics

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean</th>
<th>Median</th>
<th>Std.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Full sample</td>
<td>Good health</td>
<td>Bad health</td>
</tr>
<tr>
<td>Age</td>
<td>45.2</td>
<td>44</td>
<td>50</td>
</tr>
<tr>
<td>Good health</td>
<td>0.81</td>
<td>–</td>
<td>–</td>
</tr>
<tr>
<td>Employed</td>
<td>0.87</td>
<td>0.92</td>
<td>0.67</td>
</tr>
<tr>
<td>Retired</td>
<td>0.05</td>
<td>0.02</td>
<td>0.18</td>
</tr>
<tr>
<td>Hourly wage (€)</td>
<td>16.8</td>
<td>16.9</td>
<td>16.1</td>
</tr>
<tr>
<td>Years of education</td>
<td>12.3</td>
<td>12.4</td>
<td>11.7</td>
</tr>
<tr>
<td>Work experience</td>
<td>22.1</td>
<td>21.2</td>
<td>25.8</td>
</tr>
<tr>
<td>Savings of non-retirees (€)</td>
<td>4,450</td>
<td>4,629</td>
<td>3,578</td>
</tr>
<tr>
<td>Net wealth of non-retirees (€)</td>
<td>112,452</td>
<td>112,689</td>
<td>111,272</td>
</tr>
</tbody>
</table>

The model is estimated for the waves after the 2005 reform of the German income tax system. On January 1, 2005, a reform of the German tax system came into effect lowering the marginal tax rates and making some changes to the tax base. The focus on waves with a relatively homogeneous institutional framework facilitates the computation of the value functions. The survey years 2003 and 2004 are included to provide information for lagged variables of the model.
**Health status:** The SOEP provides annual information on individuals’ health status by both a measure of legally attested disability status and of self-assessed health (SAH). Legally attested disability is based on a medical examination and has the advantage of being comparatively objective. However, there may be a lag between the realization of a health shock and the completion of the process leading to the approval of the disability status. Moreover, this measure may not capture some forms of mental illnesses or physical impairments that are relevant when investigating the effects of bad health on economic outcomes. The eligibility criteria of the early retirement scheme are independent of the legal disability status.

I combine the objective disability measure with the subjective SAH measure that presumably captures a broader range of health problems and is found to reflect longitudinal changes in the objective health status reasonably well (Benitez-Silva and Ni, 2008). I construct a binary health measure defining good health as neither being officially disabled nor assessing own health as poor or bad. By this definition, 81% of the individuals in the sample are in good health status. I observe 904 transitions from good to bad health status and 749 transitions from bad to good health status. There are 509 cases of individuals who experience a transition from bad to good health status after having experienced a health shock in the previous period. Hence, about 56% of the bad health shocks are transitory. 466 out of the 509 cases of transitory bad health shocks are due to changes in self-rated health (i.e. from poor or bad to satisfactory, good, or very good). Hence, the legal disability status tends to be more persistent capturing many of the severe cases. For individuals who have been observed in bad health in two consecutive periods, the share of individuals who experience a transition from bad to good health status after having experienced a health shock in the previous period goes down from 56% to only 12% (240 observed transitions). The difference in the transition probabilities between transitory and more permanent health shocks is taken into account by the AR(2)-structure of the health transition equation in the model.

**Labor market:** Since part-time employment is empirically irrelevant for German males, employment behavior is only differentiated between non-employment and full-time employment. Employment is defined as working at least 20 hours per week and the median hours of work for the employees is 40.\(^9\) The share of employed individuals is insensitive to the exclusion of observations with working hours between 5 and 35.
als amounts to 87% in the sample and differs substantially by health status (92% for individuals in good health status and 67% for individuals in bad health status). Retirees make up 5% of the sample. The mean wage of employed individuals is €16.8.\textsuperscript{10} Education is measured as years of education. The SOEP constructs the years of education variable from respondents’ information on the obtained level of education and adds some time for additional occupational training. The median number is 12 (cutoff point for high/low in the model). Work experience is defined as years of full-time experience. For this reason, one year of pre-sample part-time experience is counted as half a year of full-time experience.

\textit{Wealth and saving:} Wealth information in the SOEP is fairly complete, but is contained in the survey only for the years 2002, 2007, and 2012. For consistency, I only rely on the information from the 2007 wave. In this wave, the information comprises market values of real estates, financial assets, building loan contracts, private insurances, business assets, tangible assets, consumer debts, and overall debts. I compute net wealth by combining the information on gross wealth and debts. The variable is imputed for the 2005, 2006, 2008, 2009, 2010, 2011, and 2012 waves.\textsuperscript{11} This is done using information on saving behavior and carrying forward net wealth from the year 2007 to the other survey years. For this purpose, I assume that individuals borrow at a real interest rate of 6% and receive a real interest rate of 2% on both their real and financial wealth holdings. Moreover, I take into account information on wealth losses that is provided by an additional variable. I ensure consistency of the wealth measure with the model assumptions by introducing a censoring such that individuals always have non-negative net wealth and can have at most as much net wealth as they could possibly have accumulated within the life cycle model until their respective ages. Average net wealth of non-retirees is €112,452, but the median is only €56,106. About 15% of the individuals do not have any positive net wealth.

Similarly to Fuchs-Schuendeln (2008), I define total positive saving as the sum of financial and real annual saving. Since the saving information is bottom-coded at zero

\textsuperscript{10}Some outliers with wages below the 1%-percentile or above the 99%-percentile are excluded from the sample. My results are insensitive to this choice of sample restriction.

\textsuperscript{11}Some other studies derive wealth from annual information on asset income and home ownership (e.g. Fuchs-Schuendeln and Schuendeln, 2005; Haan and Prowse, 2014). However, this measure suffers from weaknesses following from unobserved fluctuations in the returns of individual asset portfolios and from the fact that some assets do not generate any annual returns.
in the SOEP, I assume that working individuals have non-negative net saving over the period of a whole year and make assumptions on the dissaving of the unemployed and retirees. Note that if somebody takes out a loan to buy a house, this would not affect net wealth because the financial dissaving is compensated by real saving. Unemployed individuals are assumed to dissave in the case that they are not eligible for unemployment insurance benefits and fail the means test required for social assistance benefits. Retirees are assumed to buy an actuarially fair life annuity with their accumulated wealth at retirement and make no more saving choices. Appendix B provides some more details on the saving variable and the saving information in the SOEP. The median saving rate before retirement is 10% in the sample.

4 Estimation and model fit

In principle, the model can be estimated by finding the maximum of the log-likelihood function. However, it is due to the non-separability of the log-likelihood function that a stepwise maximization is impossible. Unless the starting values in the optimization algorithm are very good, a direct maximization with respect to all parameters involves considerable numerical problems and is computationally intense. For this reason, I resort to an extension of the EM algorithm that has been proposed by Arcidiacono and Jones (2003) for the estimation of finite mixture models with time-constant unobserved heterogeneity. They show how this algorithm can facilitate the estimation with little loss in efficiency by allowing for a sequential estimation of the parameters (see Appendix D for a detailed description of the estimation approach). Following Arcidiacono and Jones (2003), I use the extended EM algorithm to obtain good starting values for a subsequent efficient full information maximum likelihood procedure.

The extended EM algorithm gets close to convergence after a number of iterations that depends on the choice of starting values. I abort the algorithm after 10 iterations and use the current trail values of the parameters as initial values in a FIML estimation. While the extended EM algorithm slows down when approaching convergence, the optimization algorithm that is used for the FIML estimation converges comparatively

The starting values are shown in table 2. Many of them are simply set to 0.1 and the estimation results do not hinge on the choice of starting values.
quickly when using good starting values.\textsuperscript{13} The log-likelihood at its maximum and at the trail values from the EM algorithm after 10 iterations differ only slightly (see table 2). Using bad starting values for the FIML procedure usually results in non-convergence of the optimization algorithm. The model is estimated allowing for three unobserved types (M=3). I could not identify more than three types. The main results of the analysis are insensitive to a reduction from three to two types.

4.1 Parameter estimates

Table 2 shows the parameter estimates, the current trail values of the extended EM algorithm after 10 iterations, and the starting values used for the extended EM algorithm. In the following, I shortly discuss the estimation results:

**Utility function:** The estimates of the coefficient of relative risk aversion are 0.242, 0.704, and 0.733 for the three types. These estimates of $\rho_m$ are larger than the estimate of Rust and Phelan (1997), but smaller than the respective estimates of French (2005) and French and Jones (2011). Furthermore, the estimates of the type-specific parameter $\alpha_{2m}$ indicate substantial unobserved heterogeneity in individuals’ disutility of work (ranging from -1.729 to -0.199).

**Job offer and separation rates:** The parameter estimates of the job offer and separation rates show that both rates are affected strongly by an individual’s health status. Hence, a health shock affects the labor market risks both by lowering the probability of a job offer and by raising the probability of a job separation. Education exerts comparatively small effects on the offer and separation rates. High education goes along with both lower separation and lower offer rates. After age 50 and even more after age 55 and age 60, separation rates increase. Offer rates strongly decline after age 50 and further decline after age 60. This induces many individuals to opt for early retirement if eligible. Table 6 in Appendix E displays the rates implied by the parameter estimates.\textsuperscript{14}

\textsuperscript{13}I use an unconstrained optimization algorithm providing a numerical gradient and BHHH Hessian.

\textsuperscript{14}The parameter estimates for the utility function and for the offer and separation rates determine the net wage elasticity. At prime working age, the simulated elasticity is quite small (e.g. 0.062 at age 30-34). For these individuals, the incentive to participate in the labor market is strong and, therefore, the probability that they opt for unemployment even if they do not face a labor market friction (job separation or no job offer) is small and insensitive to small changes in the wage. At the end of the working life, the simulated net wage elasticity is substantially larger (0.148 at age 60-64). Given that about 20% of the individuals retire already before age 60 in the simulations, this elasticity even needs
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>FIML Estimates</th>
<th>EM algorithm Estimates (10 iterations)</th>
<th>Starting values (10 iterations)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Utility function:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$a_1$</td>
<td>(scaling factor)</td>
<td>2.295 (0.0513)</td>
<td>2.264</td>
<td>1</td>
</tr>
<tr>
<td>$a_{21}$</td>
<td>(work, type 1)</td>
<td>-1.729 (0.0732)</td>
<td>-1.671</td>
<td>-0.5</td>
</tr>
<tr>
<td>$a_{22}$</td>
<td>(work, type 2)</td>
<td>-0.951 (0.0538)</td>
<td>-0.926</td>
<td>-0.5</td>
</tr>
<tr>
<td>$a_{23}$</td>
<td>(work, type 3)</td>
<td>-0.199 (0.0731)</td>
<td>-0.198</td>
<td>-0.5</td>
</tr>
<tr>
<td>$\rho_1$</td>
<td>(crra, type 1)</td>
<td>0.242 (0.0316)</td>
<td>0.227</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_2$</td>
<td>(crra, type 2)</td>
<td>0.704 (0.0358)</td>
<td>0.718</td>
<td>0.5</td>
</tr>
<tr>
<td>$\rho_3$</td>
<td>(crra, type 3)</td>
<td>0.733 (0.0240)</td>
<td>0.77</td>
<td>0.5</td>
</tr>
<tr>
<td>Job offer and separation rates:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>(separation, constant)</td>
<td>-2.535 (0.1035)</td>
<td>-2.571</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi_2$</td>
<td>(separation, good health)</td>
<td>-1.26 (0.1059)</td>
<td>-1.195</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi_3$</td>
<td>(separation, high educ)</td>
<td>-0.294 (0.0936)</td>
<td>-0.347</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi_4$</td>
<td>(separation, age $\geq$ 50)</td>
<td>0.528 (0.1703)</td>
<td>1.031</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_5$</td>
<td>(separation, age $\geq$ 55)</td>
<td>-1.755 (0.1289)</td>
<td>-1.679</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi_6$</td>
<td>(separation, age $\geq$ 60)</td>
<td>1.85 (0.1397)</td>
<td>1.704</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi_7$</td>
<td>(offer, constant)</td>
<td>-1.755 (0.1289)</td>
<td>-1.679</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi_8$</td>
<td>(offer, good health)</td>
<td>-1.26 (0.1059)</td>
<td>-1.195</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi_9$</td>
<td>(offer, high educ)</td>
<td>-0.294 (0.0936)</td>
<td>-0.347</td>
<td>0.1</td>
</tr>
<tr>
<td>$\phi_{10}$</td>
<td>(offer, age $\geq$ 50)</td>
<td>0.528 (0.1703)</td>
<td>1.031</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_{11}$</td>
<td>(offer, age $\geq$ 55)</td>
<td>-1.755 (0.1289)</td>
<td>-1.679</td>
<td>0.1</td>
</tr>
<tr>
<td>Wage equation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta_{11}$</td>
<td>(constant, type 1)</td>
<td>2.124 (0.0111)</td>
<td>2.115</td>
<td>2</td>
</tr>
<tr>
<td>$\delta_{12}$</td>
<td>(constant, type 2)</td>
<td>1.771 (0.0109)</td>
<td>1.765</td>
<td>1.75</td>
</tr>
<tr>
<td>$\delta_{13}$</td>
<td>(constant, type 3)</td>
<td>1.375 (0.0111)</td>
<td>1.376</td>
<td>1.5</td>
</tr>
<tr>
<td>$\delta_2$</td>
<td>(years of education/10)</td>
<td>0.476 (0.0066)</td>
<td>0.472</td>
<td>0.5</td>
</tr>
<tr>
<td>$\delta_3$</td>
<td>((experience/10)*(educ &lt; 12))</td>
<td>0.267 (0.0055)</td>
<td>0.264</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_4$</td>
<td>((experience/10)*(educ $\geq$ 12))</td>
<td>0.293 (0.0060)</td>
<td>0.291</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_5$</td>
<td>((experience^2/100)*(educ &lt; 12))</td>
<td>0.042 (0.0012)</td>
<td>0.042</td>
<td>0.1</td>
</tr>
<tr>
<td>$\delta_6$</td>
<td>((experience^2/100)*(educ $\geq$ 12))</td>
<td>0.046 (0.0014)</td>
<td>0.045</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\phi_7$</td>
<td>(good health)</td>
<td>-0.002 (0.0038)</td>
<td>0.008</td>
<td>-0.01</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>(standard deviation)</td>
<td>0.193 (0.0008)</td>
<td>0.193</td>
<td>0.1</td>
</tr>
<tr>
<td>Health process:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\psi_{11}$</td>
<td>(constant, type 1)</td>
<td>-1.581 (0.2420)</td>
<td>-1.463</td>
<td>-1.5</td>
</tr>
<tr>
<td>$\psi_{12}$</td>
<td>(constant, type 2)</td>
<td>-1.633 (0.2348)</td>
<td>-1.622</td>
<td>-1.5</td>
</tr>
<tr>
<td>$\psi_{13}$</td>
<td>(constant, type 3)</td>
<td>-1.745 (0.2380)</td>
<td>-1.716</td>
<td>-1.5</td>
</tr>
<tr>
<td>$\psi_2$</td>
<td>(health$^-1$)</td>
<td>2.561 (0.0698)</td>
<td>2.489</td>
<td>1.5</td>
</tr>
<tr>
<td>$\psi_3$</td>
<td>(health$^-2$)</td>
<td>2.004 (0.0670)</td>
<td>1.915</td>
<td>1.5</td>
</tr>
<tr>
<td>$\psi_4$</td>
<td>(years of education/10)</td>
<td>0.414 (0.1207)</td>
<td>0.452</td>
<td>0.1</td>
</tr>
<tr>
<td>$\psi_5$</td>
<td>(wealth/100,000)</td>
<td>0.03 (0.0238)</td>
<td>0.056</td>
<td>0.1</td>
</tr>
<tr>
<td>$\psi_6$</td>
<td>(age $\geq$ 30)</td>
<td>-0.275 (0.1846)</td>
<td>-0.26</td>
<td>0.1</td>
</tr>
<tr>
<td>$\psi_7$</td>
<td>(age $\geq$ 40)</td>
<td>-0.351 (0.0908)</td>
<td>-0.267</td>
<td>0.1</td>
</tr>
<tr>
<td>$\psi_8$</td>
<td>(age $\geq$ 50)</td>
<td>-0.57 (0.0752)</td>
<td>-0.538</td>
<td>0.1</td>
</tr>
<tr>
<td>$\psi_9$</td>
<td>(age $\geq$ 60)</td>
<td>0.019 (0.1152)</td>
<td>-0.125</td>
<td>0.1</td>
</tr>
<tr>
<td>Type probabilities:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>(prob. type 1, educ &lt; 12)</td>
<td>0.194 (0.0123)</td>
<td>0.203</td>
<td>0.33</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>(prob. type 2, educ &lt; 12)</td>
<td>0.532 (0.0160)</td>
<td>0.525</td>
<td>0.33</td>
</tr>
<tr>
<td>$\gamma_3$</td>
<td>(prob. type 1, educ $\geq$ 12)</td>
<td>0.385 (0.0141)</td>
<td>0.39</td>
<td>0.33</td>
</tr>
<tr>
<td>$\gamma_4$</td>
<td>(prob. type 2, educ $\geq$ 12)</td>
<td>0.431 (0.0148)</td>
<td>0.425</td>
<td>0.33</td>
</tr>
</tbody>
</table>

Note: The indicated starting values are used for the extended EM algorithm, which is aborted after 10 iterations. Then, the current trial values of the parameters are used as starting values for the FIML estimation. Standard errors are derived from the inverse of the Hessian of the log-likelihood function at its maximum. The sample consists of 3,078 individuals and, in total, of 15,262 observations.
**Wage equation:** The estimates suggest substantial unobserved heterogeneity in the earnings capacity of individuals that is captured by the type-specific constant $\delta_{1m}$ and that is negatively correlated with the leisure preferences. The returns to education are estimated to be 4.8% per year. This estimate is a bit smaller than it usually comes out for specifications without unobserved heterogeneity. The interaction terms between work experience and education high/low suggest heterogeneous returns to experience and, thus, diverging wage profiles for the two groups. In line with the descriptive statistics, the direct effect of health status on labor productivity is small and insignificant. Of course, there is an indirect effect of past health outcomes on wages through work experience that is captured by the model.

**Health transitions:** The coefficient estimates $\psi_2$ and $\psi_3$ suggest considerable state dependence and capture the prevalence of both temporary and more persistent health shocks. Individuals who experience a health shock and do not recover after one period tend to remain in bad health status. Years of education are associated positively with an individual’s probability of good health. The estimates of $\psi_6$, $\psi_7$, $\psi_8$, and $\psi_9$ reflect the life cycle pattern of the health risks. Since the age dummies indicate an age being greater or equal than a certain threshold value, the estimated age effects are adding up over the life cycle. While the probability of a transition to good health status decreases at ages 30, 40, and 50, there is no further decrease at age 60. The effect of net wealth is estimated to be small and insignificant. Figure 3 in Appendix F displays age-specific average transition probabilities by previous periods’ health status and education (for 10 and 18 years) that are implied by the parameter estimates.

### 4.2 Synthetic sample and life cycle profiles

I check the model fit by simulating a synthetic sample of 5,000 life cycles relying on the parameter estimates. Initial conditions are drawn from the estimated distribution of unobserved types and from the observed distribution of years of education. Individuals enter the labor force at age $\max(20, 8 + \text{years of education})^{15}$ in good health, non-employed, and with zero net wealth. Choices and random transitions of state variables to be scaled up by about 25% when considering the population of individuals that are still in the labor force at age 60.

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15 This assumes that they start school at age 7 and have experienced one year of either civil or military service.
are based on the respective probabilities of the life cycle model and pseudo-random draws from the uniform distribution.

Figure 1: Comparison of average simulated outcomes and sample means

Figure 1 and figure 2 compare average simulated life cycle outcomes with age-specific means from the estimation sample. Figure 1 shows the respective comparisons for retirement, employment, gross wages, and net wealth. The age-specific fit of these outcomes is reasonably good and the model seems to capture the relevant life cycle patterns. This is also the case when comparing these outcomes for individuals in good and bad health separately (see figures 4 and 5 in Appendix G).\textsuperscript{16} Furthermore, the Gini of simulated gross lifetime earnings (0.205) is in line with the findings of Bönke, Corneo, and Lüthen (2015) that are based on administrative data.

\textsuperscript{16}I somewhat underpredict the share of retirees in bad health status before age 50. This follows from the fact that in the model young individuals in bad health status have a strong incentive to keep the option value of potentially returning to the labor force and, therefore, prefer receiving social assistance benefits. Fortunately, this does not matter for their income because the disability benefits of these individuals usually would be anyway below the consumption minimum that is guaranteed through social assistance.
Figure 2 focuses on observed and estimated shares of individuals in good health status by previous periods’ health status. In particular, a comparison of the two lower graphs demonstrates the relevance of the second order lag in the health transitions. The share of individuals in bad health who experience a transition to good health status is substantially higher if the individuals have only been in bad health status for one period. Generally, the share of individuals who recover from a health shock declines markedly with age. The comparisons suggest that the health equation is sufficiently flexible in order to capture individuals’ health risks.

5 Policy Analysis

This section investigates the health-related risks of average annual consumption and old-age poverty based on the estimated life cycle model. While the first subsection focuses on the health-related risks in the current institutional framework, the second subsection presents simulations that investigate means-tested minimum pension bene-
fits as an insurance against old-age poverty. The health-related risks are computed by comparing simulated outcomes for a representative synthetic sample of 5,000 life cycles with the respective outcomes of a simulated benchmark scenario where no health shocks occur. For this purpose, I simulate both a scenario where stochastic health shocks occur according to the estimated health process (see the previous section for details on the associated health transition rates) and a counterfactual scenario where all individuals remain in good health until they die. Furthermore, I consider a scenario where individuals experience a persistent health shock at age 40, while no health shock occurred before this age. Persistence means in this scenario that the individual remains in bad health from age 40 onwards. This provides some evidence for the substantial implications that a health shock may have when occurring at a relatively early stage in the life cycle. For comparison, I also show the results for a persistent health shock at age 60. In all scenarios, individuals expect health to evolve stochastically according to the estimated health process in future periods. The life cycle simulations are implemented as described in the previous section. Note that the model does not include private disability insurance. For this reason, a comparison of average outcomes between the scenarios indicates the risks that are uninsured by the public insurance schemes. If individuals have bought additional private disability insurance, they face a lower level of risks.

5.1 Health-related risks under current institutions

Table 3 presents average effects of health shocks on behavioral outcomes, average annual consumption, and the risk of old-age poverty. The table also displays the mean of average annual consumption and the risk of old-age poverty under stochastic health (i.e. with health shocks being realized according to the estimated health process). The first part of the table shows the effects of stochastic health shocks that follow the estimated health transitions (relative to the benchmark scenario). The second part shows the average effects of a persistent health shock at age 40 and 60 (i.e. with individuals remaining in bad health from age 40 or 60 onwards). In terms of behavioral outcomes, I present the effects on retirement age, work experience at retirement (measuring the years in full-time employment), net wealth at age 40 and net wealth at age 60. The two latter outcomes capture effects on saving behavior. I not only present the overall
Table 3: Average effects of health shocks on life-cycle outcomes by endowments

<table>
<thead>
<tr>
<th>Life-cycle outcome</th>
<th>Years of educ.&lt;12</th>
<th>Years of educ.≥12</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type 1 (10.26%)</td>
<td>Type 2 (25.50%)</td>
<td>Type 3 (13.04%)</td>
</tr>
<tr>
<td>Stochastic health shocks:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆Retirement age</td>
<td>-4.9</td>
<td>-2.4</td>
<td>-0.6</td>
</tr>
<tr>
<td>(years)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆Work experience at ret. (years)</td>
<td>-4.9</td>
<td>-3.7</td>
<td>-3.7</td>
</tr>
<tr>
<td>∆Wealth at age 40 of non-retirees (€)</td>
<td>-3,991</td>
<td>-1,598</td>
<td>-1,300</td>
</tr>
<tr>
<td>∆Wealth at age 60 of non-retirees (€)</td>
<td>-17,719</td>
<td>-21,555</td>
<td>-19,152</td>
</tr>
<tr>
<td>Mean of average annual consumption (€)</td>
<td>19,652</td>
<td>14,956</td>
<td>10,472</td>
</tr>
<tr>
<td>∆Average annual consumption (%)</td>
<td>-13.9</td>
<td>-9.9</td>
<td>-8.7</td>
</tr>
<tr>
<td>Share in old-age poverty (%)</td>
<td>1.8</td>
<td>7.0</td>
<td>31.2</td>
</tr>
<tr>
<td>∆Share in old-age poverty (pp)</td>
<td>+1.6</td>
<td>+6.0</td>
<td>+21.5</td>
</tr>
<tr>
<td>Persistent shock at age 40:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆Retirement age</td>
<td>-9.7</td>
<td>-4.2</td>
<td>-0.6</td>
</tr>
<tr>
<td>(years)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆Work experience at ret. (years)</td>
<td>-10.7</td>
<td>-10.1</td>
<td>-9.6</td>
</tr>
<tr>
<td>∆Wealth at age 60 of non-retirees (€)</td>
<td>-120,643</td>
<td>-100,984</td>
<td>-50,715</td>
</tr>
<tr>
<td>∆Average annual consumption (%)</td>
<td>-31.5</td>
<td>-29.6</td>
<td>-21.5</td>
</tr>
<tr>
<td>∆Share in old-age poverty (%)</td>
<td>+6.0</td>
<td>+35.9</td>
<td>+59.2</td>
</tr>
<tr>
<td>Persistent shock at age 60:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆Retirement age</td>
<td>-1.3</td>
<td>-1.3</td>
<td>-1.0</td>
</tr>
<tr>
<td>(years)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>∆Work experience at ret. (years)</td>
<td>-1.0</td>
<td>-1.0</td>
<td>-1.1</td>
</tr>
<tr>
<td>∆Average annual consumption (%)</td>
<td>-5.4</td>
<td>-3.5</td>
<td>-3.1</td>
</tr>
<tr>
<td>∆Share in old-age poverty (pp)</td>
<td>+0.2</td>
<td>+0.1</td>
<td>+2.9</td>
</tr>
</tbody>
</table>

Note: The average effects are simulated on the basis of 5000 life-cycles relative to a baseline scenario where no health shocks occur. All individuals are assumed to enter the labor force in good health and non-employed. Hence, they only differ – ex ante – in terms of their unobserved type \( m \in \{1, 2, 3\} \) and educational attainment (draws are taken from the estimated distribution). The simulated shares of the productivity types by education (high/low) are indicated in parenthesis.
average effect but also average effects by initial endowments that differ by unobserved type and educational attainments of the individuals. This provides some insights on heterogeneity in the health-related risks. In the following subsections, I first discuss the behavioral responses to the health shocks, then the effects on average annual consumption, and at last the effects on the risk of old-age poverty. The measures of average annual consumption and old-age poverty are discussed in the respective subsections.

5.1.1 Behavioral responses

In comparison to the benchmark scenario, stochastic health shocks induce a substantial decrease in average retirement age (-2.4 years). Average retirement age under stochastic health is about 61.7, but would have been about 64.1 in the absence of health shocks. Behavioral responses at the retirement margin are very heterogeneous ranging from -0.6 years for type 3 (low productivity) individuals with low education to -4.9 years for type 1 (high productivity) individuals with low education. The heterogeneity is mainly driven by the following mechanisms: (1) Low productivity individuals are likely to receive a disability pension that is below or just at the level of the social security minimum. Hence, it may be more beneficial for them to remain in the labor force maintaining the option value of a recovery from bad health and a return to employment. (2) High education raises the probability of a transition from bad to good health status and, thus, leads to an increase in the option value of non-employment relative to early retirement with a disability pension. (3) The higher an individual’s productivity, the higher is the opportunity cost of early retirement.

The average loss in years of employment that is due to stochastic health shocks amounts to 3.6 years. There is much less heterogeneity in the effects on life cycle employment than at the retirement margin. This follows from the fact that types of individuals that opt less often for the disability pension spend more time in unemployment instead. Heterogeneity between individuals with high and low education reflects the higher health risks of the low educated. Considering changes in net wealth at age 40 and 60 of the non-retirees, there are substantial effects. The average decrease in net wealth at age 60 is €16,575. This reflects the fact that individuals may be unable to self-insure themselves against losses in their pension claims by accumulating wealth. I only look at net wealth of non-retirees because the model assumes that individuals
buy a fair life annuity with their wealth at retirement.

Considering the scenario of a persistent health shock at age 40, the simulations suggest a very strong decrease in the average years of employment at retirement (-9.7 years) as well as in the average retirement age (-5.1 years). Furthermore, average net wealth at age 60 of the non-retirees decreases by €82,065. This suggests severe consequences for the financial situation of these individuals.

5.1.2 Average annual consumption

I assess the effects of health shocks on individuals’ consumption by considering a measure of average annual consumption. This measure is defined as the average consumption in all years of realized lifetime after an individual enters the labor force and should reflect the average standard of living. The average losses in average annual consumption that are due to stochastic health shocks indicate the magnitude of the health-related consumption risks. The simulations suggest that the losses amount on average to 9.8% and they tend to be larger for the low educated as well as for the type 1 individuals (high productivity) in percentage terms. Depending on unobserved types and level of education, average losses range between 6.7% and 13.9%. Losses can be severe if a health shock occurs at an early stage in the life cycle. The simulations suggest that a persistent health shock at age 40 induces average losses in average annual consumption of 28.8% that are uninsured by the public insurance schemes.

5.1.3 Old-age poverty

This subsection focuses on a distributional outcome examining the health-related risk of old-age poverty. For this purpose, I take the EU definition of relative poverty as a reference that defines individuals as being at the “risk of poverty” when receiving a net income below 60% of the median net equivalent income. This indicates a threshold value of €816 of monthly net income in 2005 (Statistisches Bundesamt, 2008). I use this threshold value because all nominal variables in my sample are adjusted to the purchasing power in 2005. I define individuals as poor who experience a level of monthly consumption below €816. This is somewhat more conservative than the EU definition because monthly consumption after retirement may be higher than monthly income (as individuals receive an annuity income in addition to their pension benefits). The
choice of the threshold value is inevitably a bit arbitrary. However, the overall pattern of the simulated risks of old-age poverty and the health-related changes in these risks are insensitive to variations of the threshold value.

In the simulations, stochastic health shocks raise the risk of old-age poverty by about 7 percentage points. Given a level of old-age poverty of 9.6%, this suggests that health shocks account for more than two-thirds of the overall risk. However, the effects are very heterogeneous and range from +1.6 percentage points for type 1 individuals with a high level of education to +21.5 percentage points for type 3 individuals with a low level of education. In particular a persistent health shock at an early stage in the life cycle induces a dramatic increase in the risk of old-age poverty that amounts on average to +29 percentage points and up to +59.2 percentage points for type 3 individuals with a low level of education. For the high productivity individuals even a health shock at age 40 only induces a moderate rise in the risk of old-age poverty (+4.4 percentage points for type 1 individuals with a high level of education). The findings suggest that there is a substantial risk of health-related old-age poverty that is uninsured by the public insurance schemes.

5.2 Minimum pension benefits

Policy makers may consider a policy intervention to cope with the issue of health-related old-age poverty. This can be done through policies that either improve health outcomes or provide additional insurance against the economic consequences of health shocks. I focus on the latter policy option by investigating budgetary neutral reforms that insure individuals against the risk of old-age poverty through a minimum level of pension benefits at the poverty line of €816 or above (scenarios with €900 and €1000 are also considered). In these scenarios the risk of old-age poverty is reduced to zero. However, the reform raises a concern about an increase in abuse of the disability pension and a decline in average retirement age that is due to an increased attractiveness of the disability pension (moral hazard problem). In particular, this may be the case for individuals with otherwise very low pension claims. In the model, cheating is taken into account in the sense that individuals who are in bad health may opt for the disability pension even though they are not work incapacitated (unrestricted employment choice). Since the introduction of minimum pension benefits does not affect labor market risks,
reductions in average retirement age can be attributed to an increase in cheating. There is also a countervailing effect because minimum pension benefits reduce the risk of future pension claims by making them less dependent on labor market outcomes (lower bound). This raises the option value of remaining in the labor force as opposed to claiming disability benefits for individuals who are in bad health status.

Golosov and Tsyvinski (2006) argue that disability benefits should be means-tested in order to make false claims more unattractive. The rationale behind this idea is that individuals need saving to smooth their consumption, but the more they save the more their benefits are reduced by the means test. This may prevent false claims if the benefits are not too generous relative to the wages that individuals can earn on the labor market. In the context of a pension scheme where the option of a disability pension constitutes an insurance against work disability, this idea can be applied. Hence, a means test may reduce the potential increase in abuse that is due to the introduction of minimum pension benefits and ensure that only individuals who are in need benefit from the reform (see also Pashchenko and Porapakkarm (2017) on the topic). I investigate the theoretical argument by simulating both means-tested and non-means-tested schemes of the minimum pension benefits.

The schemes are set up as follows. Individuals’ net pension benefits are raised either by (1) the difference between the original net pension benefits and the minimum level (non-means-tested scheme) or by (2) the difference between the original net pension benefits plus net annuity income and the minimum level (means-tested scheme). The latter scheme accounts for net annuity income because the model assumes that individuals buy a fair life annuity with their accumulated wealth at retirement. Since the increase in the pension level is likely to raise the government’s budget deficit (due to additional pension expenditures and a decrease in tax payments and social security contributions), I ensure budget neutrality by raising the pension contribution rate. This is implemented by iteratively adjusting the pension contribution rate in the simulations until the following equality holds in the simulated sample:

$$\text{BD}_{before} = \text{FS}_{after} + \sum_{n=1}^{N} \sum_{t=1}^{T} \left[ \text{rate}_{before} + \tau \times \text{wage}_{after} - \text{pension benefits}_{after} \right] p(t|1, s_{nt})$$

where $\text{BD}_{before}$ is the budget deficit prior to the reform, $\text{FS}_{after}$ is the fiscal surplus...
after the reform, rate\textsubscript{before} is the original pension contribution rate (19.5\%), \( \tau \) is the adjustment to the pension contribution rate, wage\textsubscript{after} is the gross wage after the reform, and pension benefits\textsubscript{after} is the pension benefits (old-age pension plus disability pension and including minimum pensions) after the reform. \( p(t|1, s_{nt}) \) is the probability of survival until period \( t \), given survival until period 1. These probabilities are also used when computing the fiscal surplus. Iteratively adjusting \( \tau \) ensures budget neutrality while accounting for behavioral responses. No discounting is applied in the equation because all benefits are financed on a pay-as-you-go basis. This assumes that the demographic composition remains constant across birth cohorts.

Table 4 displays average effects of the introduction of minimum pension benefits on behavioral outcomes, net pension returns, and average annual consumption by endowments. The net pension returns are defined as the difference between the sum of net pension benefits and pension contributions. All other outcomes are defined as in the previous subsection. The respective results for schemes with a minimum pension of €900 and €1000 are presented in tables 8 and 9 in Appendix I. In the following subsections, I will focus on the scheme with the minimum pension at the poverty line of €816 because the interpretations apply analogously to the schemes with higher minimum pensions. It is, however, insightful to consider how higher levels of minimum pensions amplify the effects under the means-tested and the non-means-tested scheme.

When introducing minimum pensions of €816, budget neutrality requires a rise in the pension contribution rate of 1.443 percentage points in the non-means-tested scheme and of 0.298 percentage points in the means-tested scheme.

5.2.1 Non-means-tested scheme

The introduction of minimum pension benefits of €816 as a non-means-tested scheme induces a decrease in average retirement of 0.44 years and in average work experience at retirement of 0.1 years. The simulations suggest a severe moral hazard problem for the type 3 individuals (low productivity) who reduce their average retirement age by 1.65 years when having a low level of education. The increase in the pension contribution rate by 1.443 percentage points that ensures the budget neutrality only accounts for

\footnote{This is the expected sum of taxes (income plus capital taxes) plus health and unemployment insurance contributions minus the expected sum of unemployment insurance and social assistance benefits.}
Table 4: Average effects of minimum pension benefits at the poverty line (€816 per month)

<table>
<thead>
<tr>
<th>Life-cycle outcome</th>
<th>Years of educ.&lt;12</th>
<th></th>
<th>Years of educ.≥12</th>
<th></th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type 1</td>
<td>Type 2</td>
<td>Type 3</td>
<td>Type 1</td>
<td>Type 2</td>
</tr>
<tr>
<td></td>
<td>(10.26%)</td>
<td>(25.50%)</td>
<td>(13.04%)</td>
<td>(19.82%)</td>
<td>(21.26%)</td>
</tr>
<tr>
<td>ΔRetirement age</td>
<td>+0.05</td>
<td>-0.29</td>
<td>-1.65</td>
<td>-0.15</td>
<td>-0.14</td>
</tr>
<tr>
<td>(years)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔWork experience</td>
<td>+0.00</td>
<td>-0.08</td>
<td>-0.37</td>
<td>-0.03</td>
<td>-0.02</td>
</tr>
<tr>
<td>at ret. (years)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔWealth at age 40 of</td>
<td>-2,166</td>
<td>-1,014</td>
<td>-595</td>
<td>-1,515</td>
<td>-853</td>
</tr>
<tr>
<td>non-retirees (€)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔWealth at age 60 of</td>
<td>-3,473</td>
<td>+33</td>
<td>-1,183</td>
<td>+2,221</td>
<td>-1,600</td>
</tr>
<tr>
<td>non-retirees (€)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔNet pension returns (€)</td>
<td>-17,884</td>
<td>-3,751</td>
<td>+36,967</td>
<td>-18,536</td>
<td>-9,067</td>
</tr>
<tr>
<td>Fraction receiving min.</td>
<td>13.5</td>
<td>27.5</td>
<td>76.9</td>
<td>11.5</td>
<td>17.7</td>
</tr>
<tr>
<td>pension benefits (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔAverage annual consumption (€)</td>
<td>-184</td>
<td>-15</td>
<td>+541</td>
<td>-216</td>
<td>-90</td>
</tr>
<tr>
<td>ΔAverage annual consumption (%)</td>
<td>-0.9</td>
<td>-0.1</td>
<td>+5.2</td>
<td>-0.8</td>
<td>-0.5</td>
</tr>
</tbody>
</table>

Means-tested scheme:

| ΔRetirement age (years) | +0.01 | +0.09 | +0.01 | -0.01 | +0.05 | +0.04 | +0.04 |
| ΔWork experience at ret. (years) | +0.00 | -0.02 | -0.10 | -0.01 | +0.01 | -0.10 | -0.03 |
| ΔWealth at age 40 of non-retirees (€) | -857 | -689 | -643 | -649 | -573 | -890 | -688 |
| ΔWealth at age 60 of non-retirees (€) | -2,383 | -2,633 | -1,299 | -1,055 | -1,514 | -2,027 | -1,815 |
| ΔNet pension returns (€) | -3,656 | -2,061 | +7,870 | -3,952 | -2,272 | +5,497 | -612 |
| Fraction receiving min. pension benefits (%) | 11.1 | 11.2 | 28.6 | 9.2 | 14.8 | 20.8 | 14.8 |
| ΔAverage annual consumption (€) | -46 | -43 | +94 | -58 | -41 | +35 | -20 |
| ΔAverage annual consumption (%) | -0.2 | -0.3 | +0.9 | -0.2 | -0.2 | +0.2 | -0.1 |

Note: The average effects are simulated based on 5000 life cycles. All individuals are assumed to enter the labor force in good health and non-employed. Hence, they only differ – ex ante – in terms of their unobserved type \(m \in \{1, 2, 3\}\) and educational attainment (draws are taken from the estimated distribution). The simulated shares of the productivity types by education (high/low) are indicated in parenthesis. Budget neutrality is ensured by adjusting the pension contribution rate. It has been raised by 1.443 percentage points in the non-means-tested scheme and by 0.298 percentage points in the means-tested scheme.

About 20% of the negative employment effect of the reform. Hence, the behavioral response is not driven by the specific scheme. The decrease in retirement age is due to
the increased generosity of the pension system that induces individuals to retire earlier even if their labor market choices are unrestricted. Furthermore, there is a crowding-out effect on private saving. Average net wealth of non-retirees is reduced by €1,133 at age 40 and by €566 at age 60. However, the three types are affected differentially and there is a strong countervailing effect because the minimum pension benefits replace the means-tested social assistance benefits for retirees. While all types of individuals reduce average net wealth at age 40, the countervailing effect even leads to an increase in average net wealth at age 60 for certain types.

Average net pension returns decrease because all losses in public revenue and the additional public pension expenditures are compensated by the increase in the pension contribution rate. The simulated changes in average net pension returns of the different types of individuals suggest a substantial redistribution from the high productivity to the low productivity individuals via the pension system. While type 1 individuals with high education loose on average €18,536, type 3 individuals with low education gain on average €36,967. This is due to the fact that low productivity types earn low income and, consequently, are much more likely than high productivity types to accumulate pension claims until retirement that are below the minimum pension threshold. Furthermore, the increase in the pension contribution rate leads to a larger increase in the absolute pension contributions of individuals with a high income. The gains and losses as well as the changes in employment and saving behavior translate into changes in average annual consumption. While type 3 individuals experience a substantial rise in their average annual consumption (€ +541 for the low educated and € +461 for the high educated), type 1 individuals face on average a decrease in their annual consumption possibilities. The overall slight increase in average annual consumption follows from the crowding-out effect on private saving.

5.2.2 Means-tested scheme

Considering the means-tested scheme, the simulations only indicate a small decrease in average work experience at retirement (-0.03 years) and even a slight increase in average retirement age (+0.04 years). Even type 3 individuals decrease their average work experience at retirement only by 0.1 years. Individuals with low education and a low productivity type have on average a net wealth of about €66,000 at age 60.
and, consequently, are also penalized by a means test. These results suggest that the means test mitigates the moral hazard problem substantially. Hence, the fact that annuitized wealth is now accounted for when low pensions are raised to the minimum level reduces the increase in generosity of the pension system, while still insuring against the risk of old-age poverty. Since individuals need saving to smooth their consumption, false claims of disability pensions and, generally, early retirement do not become more attractive after the introduction of the minimum pension benefits. Most individuals who work in the absence of this reform keep doing so when means-tested minimum pension benefits are introduced. The slight positive effect on retirement age is stemming from the above described countervailing effect that is associated with a rise in the option value of remaining in the labor force. For all types of individuals, there is a crowding-out effect on private saving leading to a decrease in average net wealth of the non-retirees of €688 at age 40 and of €1,815 at age 60. Hence, this crowding-out effect is smaller at age 40 and larger at age 60 than for the non-means-tested scheme.

Average net pension returns decrease and there is a similar pattern of redistribution from the high productivity to the low productivity individuals as for the non-means-tested scheme. However, the level of redistribution is much lower. While type 1 individuals with high education loose on average €3,952, type 3 individuals with low education gain on average €7,870. Again, these gains and losses as well as the changes in employment and saving behavior are reflected by the changes in average annual consumption. Overall, the simulations suggest that a means test allows mitigating the moral hazard problem of minimum pension benefits substantially and reduces the redistribution between different types of individuals in the population.

6 Robustness 1: annuities account for longevity differences by health and education

In this robustness check, I assume that individuals can buy a fair annuity at retirement that accounts for differences in the longevity risk by health status and education (high/low). The simulations are based on the parameter estimates from the baseline specification where annuities are calculated using average age-specific survival rates, but the results only change marginally when the model is also estimated for the alter-
native specification. Table 10 in Appendix K shows the results for the health-related life cycle risks based on the alternative specification.\textsuperscript{18}

The behavioral response at the retirement margin to the realization of health risks increases on average by about one fourth. This is due to individuals accounting for their higher annuity income for a given level of wealth when being in bad health status and making their retirement decision. While the results on old age poverty and on the minimum benefits are quite robust to the change in specification, the health-related consumption risk (with respect average annual consumption) decreases substantially (from 9.8\% to 4.1\%). This is an interesting additional result. It suggests that annuity markets that account for differences in the longevity risk by health status can effectively reduce the health-related consumption risks. While this is a best-case scenario (given the issues of annuity markets in the real world), the simulations provide a benchmark showing the potential of this mechanism. However, the poverty risk only slightly decreases because the poor individuals cannot accumulate a lot of wealth anyway and, thus, barely benefit from a higher annuity income.

7 Robustness 2: alternative utility specification and no offer and separation rates

I have also estimated a specification where offer rates are set to 1 and separation rates are set to 0. Instead health, education, and age are interacted with the disutility of work inside the CRRA preferences (similar to e.g. French, 2005):

$$
\mathbb{U}(s_{nt}, d_{nt}) = \alpha_1 \left[ c(s_{nt}, d_{nt}) \times \exp(\alpha_{2n} \times \text{work}(d_{nt})) \right]^{(1-\rho_m)} - 1 + \epsilon_{nt}(d_{nt})
$$

$$
\alpha_{2n} = a_{1m} + a_2 \text{educ}_{nt}^{12+} + a_3 \text{health}_{nt} + a_4 \text{age}_{nt}^{50+} + a_5 \text{age}_{nt}^{55+} + a_6 \text{age}_{nt}^{60+}
$$

In this specification, health is estimated to exert a substantial effect on the utility of working individuals. Table 10 in Appendix K shows the results for the health-related life-cycle risks based on the alternative specification.\textsuperscript{19}

Except for the level of health-related consumption risks (about half the magnitude), the main findings of the paper are robust to this change in specification. However, the

\textsuperscript{18}Results for the simulation of the minimum pension benefits differ only marginally from the ones for the baselines specification and can be obtained from the author upon request.

\textsuperscript{19}The parameters of the alternative utility function can be obtained from the author upon request.
health-related consumption risks are smaller because this specification does not directly account for the persistence of employment states and the differential health effects on transitions in and out of unemployment (as is done through offer and separation rates). Consequently, individuals can compensate more easily for health-related periods of unemployment by accumulating more wealth through precautionary saving and by working more at other times in the life cycle (e.g., by postponing retirement). But even in this specification, a more effective self-insurance is only possible for individuals with high or medium productivity.

8 Conclusion

This paper analyzes health-related risks of consumption and old-age poverty on the basis of a dynamic life cycle model of health risks, employment, early retirement, and wealth accumulation. Since the analysis abstracts from spousal behavior and income, the simulated risks can be interpreted as the risks of singles or as the upper bound for couple households when assuming that spousal behavior mitigates the consequences of a health shock. A policy analysis investigates a budgetary neutral introduction of minimum pension benefits as insurance against old-age poverty. Following a theoretical argument of Golosov and Tsyvinski (2006) that a means test may prevent false claims of benefits, I compare means-tested with non-means-tested schemes.

The simulations suggest that health shocks reduce average years of employment by 3.6 years and average retirement age by 2.4 years. The average decrease in net wealth at age 60 is about €16,500 reflecting the fact that individuals may be unable to self-insure themselves against health-related losses in their pension claims by accumulating wealth. The effects at the behavioral margins translate into average losses in average annual consumption of about 10% that are uninsured by the public insurance schemes. The risk is larger for the low educated and for the high productivity types. Furthermore, health shocks are found to account for more than two-thirds of the cases of old-age poverty, where the magnitude of the risk and the health-related changes in this risk depend substantially on an individual’s endowments and may be sizable. In particular, a persistent health shock at an early stage of the life cycle may induce severe economic consequences.
This motivates the policy analysis of minimum pension benefits at the poverty line of €816, where the risk of old-age poverty is reduced to zero. The simulations suggest that the introduction of a non-means-tested scheme induces a decrease in average retirement age of 0.44 years and in average work experience at retirement of 0.1 years. For low productivity types the moral hazard problem is severe. Furthermore, the reform induces a crowding-out effect on private saving and substantial redistribution from high to low productivity types via the pension system. When considering the means-tested scheme, the results only indicate a small decrease in average work experience at retirement and even a slight increase in the average retirement age. While the pattern of redistribution is similar to the non-means-tested scheme, the level is much lower. Overall, the simulations suggest that a means test mitigates the moral hazard problem of minimum pension benefits substantially. This finding is likely to apply to other countries with similar institutions.

Additional simulations show that annuity markets that account for differences in the longevity risk by health status can effectively reduce the consumption risks, but only slightly decrease health-related old-age poverty.
Appendix

A Institutional framework

Individuals make decisions within the framework of the German tax and transfer system and statutory pension insurance scheme. The life cycle model incorporates the main features of these institutions. In the following, I outline the rules and regulations that are applied when computing the individuals’ net income. Unemployment insurance, social assistance, and disability pension are of particular relevance. The model abstracts from family-related transfers and applies (consistently with this assumption) the rules for single households.

A.1 Taxes, contributions, and transfers

Employed individuals have to pay income tax on both their gross wages and capital income. The income tax rate increases with an individual’s taxable income and is payable on all gross income in excess of the income tax allowance. Moreover, individuals pay mandatory social security contributions for health, pension, and unemployment insurance. Social security contributions are paid at a constant rate on gross income above a lower limit and below an earnings cap, where half of the contributions are paid by the employer. The rates are 19.5% for pension insurance, 17% for health insurance including long-term care insurance, and 6.5% for unemployment insurance. Hence, the contributions of the employees amount to 21.5% of the gross wage.

If individuals are unemployed, they are eligible for either unemployment insurance benefits or means-tested social assistance benefits, where the former is proportional to the last net earnings (60% for single households) and the latter ensures a minimum income that does not depend on the individual’s employment history. Retirees with pension benefits below the social security minimum, may also be eligible for means-tested social assistance benefits. Social assistance benefits are tested against new wealth, net pension benefits, and annuity income. An amount of €10,000 is exempted from the means test. The exemption level of €10,000 is assumed because the actual rules are very complicated and enforcement of these rules is unobserved. The benefits comprise a basic amount plus the costs of rent and energy consumption. In 2005, the basic monthly amount was €345 in West Germany. Individuals are assumed to receive €300
for rent and €50 for energy consumption. While unemployment insurance benefits are paid for an entitlement period only, social assistance benefits are paid indefinitely. The entitlement period of unemployment insurance benefits differs by age and employment history. I assume that all the individuals are entitled for one year of unemployment insurance benefits after becoming unemployed. Similarly to Adda, Dustmann, Meghir, and Robin (2013), this assumption avoids an increase in the state space.

A.2 Statutory pension insurance scheme

The regular retirement age of the individuals is 65 years. After retirement, individuals receive public pension benefits. The benefits are a deterministic function of the accumulated weighted pension points that reflect an individual’s employment and wage history. This leads to dynamic incentives that can be captured by a DPDC framework. An individual accumulates one pension point for each year of employment, which is given a weight of \( \frac{w_{age, nt}}{w_{age, t}} \), where \( w_{age, nt} \) is individual n’s gross wage in year t and \( w_{age, t} \) is the average gross wage in year t. There is a year-specific cap on pension point weights, which has been approximately 2 during the observation period. Individuals obtain additional pension points for military / civil service and during periods of unemployment. Individuals who receive unemployment insurance benefits also receive pension points based on 80% of their last wage. Pension points for individuals who receive social assistance benefits are negligible.

The early retirement option constitutes an insurance against work disability. Eligibility depends on age, employment history, and health status. Individuals with a sufficiently long employment history (more than 35 years of work) are eligible for early retirement if aged 63 or over. Unemployed individuals are eligible if aged 60 or over. Börsch-Supan (2001) points out that unemployment as a transition to early retirement is likely to be endogenous and to be a strategic variable of both the employer and employee. Therefore, the model assumes that all individuals can retire upon reaching

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\(^{20}\)Between 2012 and 2029, the regular pension age will be gradually increased from 65 to 67 years for the cohorts born after 1954. The analysis abstracts from this reform.

\(^{21}\)In 2005, the value of one pension point amounted to monthly benefits of €26.13 in West Germany. Since 2005, pensioners must pay income tax on 50% of their pension benefits. Until 2040, this share will gradually increase up to 100%.

\(^{22}\)I assume that individuals can receive pension points related to unemployment benefits for at most two years.
Individuals who are in sufficiently poor health status (work disability) can opt for early retirement even before age 60. They receive pension benefits as if they had worked until age 60 (disability pension). However, this requires a medical examination that is performed by a physician from the statutory pension insurance scheme. While it is hard to believe that individuals who actually are in good health status can easily pass the examination, it appears to be likely that individuals who are in bad health status - but not work incapacitated - can pass by exaggerating their health problems. The model assumes that individuals who are in bad health status can opt for early retirement no matter whether they are work incapacitated or not. For each year of early retirement before the age of 65, 63 for individuals in bad health status, a penalty (actuarial adjustment) of 3.6% is applied on the pension benefits. The penalty is applied up to a maximum of 18% of the pension benefits, where this ceiling is reduced to 10.08% for individuals in bad health status. Hence, the disability pension is more generous than the regular old-age pension.

B Saving variable

I define total positive saving as the sum of financial and real saving and make assumptions on the dissaving of the unemployed and retirees (dissaving is unobserved in the SOEP). The SOEP participants indicate their financial saving annually by answering a question about the “usual” amount of monthly saving. Question: “Do you usually have an amount of money left over at the end of the month that you can save for larger purchases, emergency expenses or to acquire wealth? If yes, how much?” This measure should be unaffected by seasonal fluctuations. Real saving is defined as annual amortization payments. Since the SOEP question asks for the sum of amortization and interest payments, the share of interest payments must be derived from information on the amount of debts. This is done under the assumption that individuals borrow at a real interest rate of 6%. Unemployed individuals are assumed to dissave if they are not eligible for unemployment insurance benefits and fail the means test that is required for social assistance benefits. These individuals receive a level of consumption that corresponds to the minimum income level (social assistance benefits) that is deducted from their net wealth. At retirement, individuals are assumed to dissave all their ac-
cumulated net wealth by buying a fair life annuity. The annuity income is calculated on the basis of the estimated average age-specific survival rates and by assuming a real interest rate of 2%.

C Data validation

Table 5: Validation of estimation sample (SOEP, survey years 2005-2012)

<table>
<thead>
<tr>
<th>Statistics</th>
<th>SOEP sample</th>
<th>Data from federal statistical office</th>
<th>Pension data</th>
<th>Boenke et al. (2015)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-time employment rate of employees (%)</td>
<td>91.8</td>
<td>91</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of individuals retiring at age &lt; 50</td>
<td>5</td>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of individuals retiring at age 50–54</td>
<td>4.9</td>
<td>4.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of individuals retiring at age 55–59</td>
<td>7.9</td>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of individuals retiring at age 60–62</td>
<td>17.7</td>
<td>21.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of individuals retiring at age 63–64</td>
<td>20.3</td>
<td>21.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Share of individuals retiring at age ≥ 65</td>
<td>44.2</td>
<td>39.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average hourly gross wage of employees (€)</td>
<td>16.8</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gini of cross-sectional gross wage distribution</td>
<td>0.309</td>
<td>0.336</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gini of average gross wage distribution</td>
<td>0.209</td>
<td>0.212</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The statistics refer to West German male individuals who are in the labor force (have finished education and have not yet retired), are not self-employed, no civil servants, and not institutionalized. The statistics from the federal statistical office have been downloaded at https://www.destatis.de/DE/Startseite.html (accessed on 14.7.2018). The pension data is stemming from Deutsche Rentenversicherung (2015). The Ginis from Bönke, Corneo, and Lüthen (2015) have been computed using data from the German pension insurance and refer to the youngest birth cohort in their study (1949 cohort). Since the SOEP sample includes birth cohorts 1941 to 1990 the Ginis need to be compared cautiously. Further, the average wage in the SOEP sample has been computed for the survey years 2005 to 2012, while the respective Gini in Bönke, Corneo, and Lüthen (2015) refers to lifetime income. Overall, the comparison supports the representativeness of the SOEP sample.
D   EM algorithm and FIML

By Bayes rule, the conditional probability Π_{mn} that individual n is of type m, given the observed choices and the parameters that are contained in θ_U, φ, θ_h, θ_w, and γ (posterior type probability), is given by

\[ Π_{mn} = \frac{π_{mn}(γ) \prod_{t=1}^{T} L_m(d_{nt}|θ_U, φ, θ_h, θ_w, s_{nt})L_m(health_{nt}|θ_h, s_{nt}, s_{nt-1})L_m(wage_{nt}|θ_w, s_{nt})}{\sum_{m=1}^{M} π_{mn}(γ) \prod_{t=1}^{T} L_m(d_{nt}|θ_U, φ, θ_h, θ_w, s_{nt})L_m(health_{nt}|θ_h, s_{nt}, s_{nt-1})L_m(wage_{nt}|θ_w, s_{nt})} \]

Using the conditional type probabilities, the following additively separable expected log-likelihood function can be derived:

\[ \sum_{n=1}^{N} \sum_{m=1}^{M} \sum_{t=1}^{T} Π_{mn} \left( \log(L_m(d_{nt}|θ_U, φ, θ_h, θ_w, s_{nt})) + \log(L_m(health_{nt}|θ_h, s_{nt}, s_{nt-1})) + \log(L_m(wage_{nt}|θ_w, s_{nt})) \right) \]

The EM algorithm reintroduces additive separability at the maximization step. Starting with arbitrary initial values, the maximum of the log-likelihood function can be found by maximizing iteratively the expected log-likelihood function, then using the estimates of θ_U, φ, θ_h, and θ_w to estimate γ by maximizing the log-likelihood function conditionally on these parameter estimates, and finally using all the estimated parameters for updating the posterior type probabilities. Subsequently, the expected log-likelihood function is maximized again. Iterating on these steps until convergence yields the maximum of the log-likelihood function (for formal proofs, see Boyles, 1983; Wu, 1983). The additive separability of the expected log-likelihood function allows a sequential maximization. This is done by first estimating θ_h and θ_w, and then taking these estimates as given in a maximization of the expected log-likelihood function with respect to θ_U. Arcidiacono and Jones (2003) show that such an extension of the EM algorithm produces consistent parameter estimates. While this estimation approach reduces the computational burden, the parameter estimates are inefficient and the estimation of standard errors is complicated because the computational time makes the use of bootstrapping methods impractical. Therefore, I follow a recommendation by Arcidiacono and Jones (2003) and use the EM algorithm to obtain good starting values for a subsequent FIML procedure.
## E  Offer and separation rates

Table 6: Job offer and separation rates by education (high/low), health, and age

<table>
<thead>
<tr>
<th>Offer rates (%)</th>
<th>Age &lt; 50</th>
<th>50 ≥ Age &lt; 55</th>
<th>55 ≥ Age &lt; 60</th>
<th>Age ≥ 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education &lt; 12 years</td>
<td>Bad health</td>
<td>9.7</td>
<td>1.6</td>
<td>2.2</td>
</tr>
<tr>
<td></td>
<td>Good health</td>
<td>54.6</td>
<td>15.0</td>
<td>20.5</td>
</tr>
<tr>
<td>Education ≥ 12 years</td>
<td>Bad health</td>
<td>5.2</td>
<td>0.8</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>Good health</td>
<td>38.0</td>
<td>8.3</td>
<td>11.6</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Separation rates (%)</th>
<th>Age &lt; 50</th>
<th>50 ≥ Age &lt; 55</th>
<th>55 ≥ Age &lt; 60</th>
<th>Age ≥ 60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Education &lt; 12 years</td>
<td>Bad health</td>
<td>8.9</td>
<td>12.0</td>
<td>14.2</td>
</tr>
<tr>
<td></td>
<td>Good health</td>
<td>2.9</td>
<td>4.0</td>
<td>4.8</td>
</tr>
<tr>
<td>Education ≥ 12 years</td>
<td>Bad health</td>
<td>6.4</td>
<td>8.7</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>Good health</td>
<td>2.0</td>
<td>2.8</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Note: The rates are computed based on the parameter estimates of the FIML estimation.

## F  Health transitions

Figure 3: Estimated probabilities of good health status by previous periods’ health status and education

![Health transitions](image)

- health good in t−1 and good in t−2
- health good in t−1 and bad in t−2
- health bad in t−1 and good in t−2
- health bad in t−1 and bad in t−2
G  Model fit by health status

Figure 4: Average simulated outcomes and sample means for good health status

Figure 5: Average simulated outcomes and sample means for bad health status
H Distributional outcomes

Table 7: Effects of health shocks and of minimum pension benefits on distributional outcomes

<table>
<thead>
<tr>
<th></th>
<th>Baseline Specification</th>
<th>Stochastic health shocks</th>
<th>Minimum pension at poverty line (no means test)</th>
<th>Minimum pension at poverty line (means test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gini*100 of average annual consumption</td>
<td>19.08</td>
<td>+0.84</td>
<td>-1.13</td>
<td>-0.22</td>
</tr>
<tr>
<td>90/10 ratio of average annual consumption</td>
<td>2.553</td>
<td>+0.109</td>
<td>-0.241</td>
<td>-0.055</td>
</tr>
</tbody>
</table>

Note: The effects are simulated based on 5000 life cycles relative to a baseline scenario. All individuals are assumed to enter the labor force in good health and non-employed. Hence, they only differ – ex ante – in terms of their unobserved type $m \in \{1, 2, 3\}$ and educational attainment (draws are taken from the estimated distribution). The 90/10 ratio indicates the ratio of average annual consumption between the individual at the 90th percentile and at the 10th percentile.
## I Alternative minimum pension schemes

Table 8: Average effects of introduction of minimum pension benefits of €900

<table>
<thead>
<tr>
<th>Life-cycle outcome</th>
<th>Years of educ.&lt;12</th>
<th>Years of educ.≥12</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type 1 (10.26%)</td>
<td>Type 2 (25.50%)</td>
<td>Type 3 (13.04%)</td>
</tr>
<tr>
<td>ΔRetirement age</td>
<td>-0.13</td>
<td>-0.73</td>
<td>-2.43</td>
</tr>
<tr>
<td>ΔWork experience</td>
<td>-0.05</td>
<td>-0.15</td>
<td>-0.59</td>
</tr>
<tr>
<td>ΔWealth at age 40</td>
<td>-3,152</td>
<td>-1,563</td>
<td>-1,239</td>
</tr>
<tr>
<td>ΔWealth at age 60</td>
<td>-1,206</td>
<td>+3,648</td>
<td>+1,276</td>
</tr>
<tr>
<td>ΔNet pension</td>
<td>-27,499</td>
<td>-1,806</td>
<td>+55,770</td>
</tr>
<tr>
<td>returns (€)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction receiving</td>
<td>12.9</td>
<td>30.9</td>
<td>98.2</td>
</tr>
<tr>
<td>ΔAverage annual</td>
<td>-282</td>
<td>+41</td>
<td>+779</td>
</tr>
<tr>
<td>consumption (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔAverage annual</td>
<td>-1.4</td>
<td>+0.3</td>
<td>+7.4</td>
</tr>
<tr>
<td>consumption (%)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Non-means-tested scheme:**

**Means-tested scheme:**

*Note:* The average effects are simulated on the basis of 5000 life-cycles. All individuals are assumed to enter the labor force in good health and non-employed. Hence, they only differ – ex ante – in terms of their unobserved type \( m \in \{1, 2, 3\} \) and educational attainment (draws are taken from the estimated distribution). The simulated shares of the productivity types by education (high/low) are indicated in parenthesis. Budget neutrality is ensured by adjusting the pension contribution rate. It has been raised by 2.515 percentage points in the non-means-tested scheme and by 0.585 percentage points in the means-tested scheme.
Table 9: Average effects of introduction of minimum pension benefits of €1000

<table>
<thead>
<tr>
<th>Life-cycle outcome</th>
<th>Years of educ.&lt;12</th>
<th>Years of educ.≥12</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type 1 (10.26%)</td>
<td>Type 2 (25.50%)</td>
<td>Type 3 (13.04%)</td>
</tr>
<tr>
<td>ΔRetirement age (years)</td>
<td>-0.27</td>
<td>-1.37</td>
<td>-3.60</td>
</tr>
<tr>
<td>ΔWork experience at ret. (years)</td>
<td>-0.06</td>
<td>-0.29</td>
<td>-0.98</td>
</tr>
<tr>
<td>ΔWealth at age 40 of non-retirees (€)</td>
<td>-4,703</td>
<td>-2,279</td>
<td>-2,051</td>
</tr>
<tr>
<td>ΔWealth at age 60 of non-retirees (€)</td>
<td>+2,416</td>
<td>+5,694</td>
<td>+5,733</td>
</tr>
<tr>
<td>ΔNet pension returns (€)</td>
<td>-44,229</td>
<td>+2,211</td>
<td>+81,650</td>
</tr>
<tr>
<td>Fraction receiving min. pension benefits (%)</td>
<td>15.3</td>
<td>49.7</td>
<td>99.2</td>
</tr>
<tr>
<td>ΔAverage annual consumption (€)</td>
<td>-432</td>
<td>+124</td>
<td>+1094</td>
</tr>
<tr>
<td>ΔAverage annual consumption (%)</td>
<td>-2.2</td>
<td>+0.8</td>
<td>+10.4</td>
</tr>
</tbody>
</table>

Means-tested scheme:

<table>
<thead>
<tr>
<th></th>
<th>Type 1 (10.7%)</th>
<th>Type 2 (13.5%)</th>
<th>Type 3 (31.6%)</th>
<th>Type 1 (9.5%)</th>
<th>Type 2 (14.9%)</th>
<th>Type 3 (23.2%)</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔRetirement age (years)</td>
<td>+0.10</td>
<td>+0.22</td>
<td>-0.47</td>
<td>-0.07</td>
<td>+0.05</td>
<td>-0.13</td>
<td>-0.01</td>
</tr>
<tr>
<td>ΔWork experience at ret. (years)</td>
<td>+0.00</td>
<td>-0.03</td>
<td>-0.37</td>
<td>-0.02</td>
<td>+0.00</td>
<td>-0.22</td>
<td>-0.08</td>
</tr>
<tr>
<td>ΔWealth at age 40 of non-retirees (€)</td>
<td>-2,396</td>
<td>-2,045</td>
<td>-2,065</td>
<td>-1,907</td>
<td>-1,704</td>
<td>-2,089</td>
<td>-1,988</td>
</tr>
<tr>
<td>ΔWealth at age 60 of non-retirees (€)</td>
<td>-8,671</td>
<td>-4,687</td>
<td>-1,099</td>
<td>-1,275</td>
<td>-2,571</td>
<td>-3,265</td>
<td>-3,335</td>
</tr>
<tr>
<td>ΔNet pension returns (€)</td>
<td>-13,388</td>
<td>-5,270</td>
<td>+27,727</td>
<td>-13,535</td>
<td>-6,107</td>
<td>+18,960</td>
<td>-1,257</td>
</tr>
<tr>
<td>Fraction receiving min. pension benefits (%)</td>
<td>10.7</td>
<td>13.5</td>
<td>31.6</td>
<td>9.5</td>
<td>14.9</td>
<td>23.2</td>
<td>16.0</td>
</tr>
<tr>
<td>ΔAverage annual consumption (€)</td>
<td>-155</td>
<td>-85</td>
<td>+295</td>
<td>-194</td>
<td>-117</td>
<td>+188</td>
<td>-43</td>
</tr>
<tr>
<td>ΔAverage annual consumption (%)</td>
<td>-0.8</td>
<td>-0.6</td>
<td>+2.8</td>
<td>-0.7</td>
<td>-0.6</td>
<td>+1.2</td>
<td>-0.2</td>
</tr>
</tbody>
</table>

Note: The average effects are simulated on the basis of 5000 life-cycles. All individuals are assumed to enter the labor force in good health and non-employed. Hence, they only differ – ex ante – in terms of their unobserved type $m \in \{1, 2, 3\}$ and educational attainment (draws are taken from the estimated distribution). The simulated shares of the productivity types by education (high/low) are indicated in parenthesis. Budget neutrality is ensured by adjusting the pension contribution rate. It has been raised by 4.235 percentage points in the non-means-tested scheme and by 1.053 percentage points in the means-tested scheme.
J Heterogeneous survival rates

I use reported cases of death in the SOEP in order to estimate survival rates from age 20 to 99 by education (high/low) and by health status based on a proportional hazard model. In the hazard model, I assume a Weibull distribution, but the results are insensitive to changes in the distributional assumption. Since panel attrition is an issue here, I resort to information from a follow-up survey that has been conducted for the last time in 2008 by the SOEP to collect information on the cases of death in the households that left the SOEP study (because individuals are likely to leave the study before they die). The estimates are based on an unbalanced panel of male individuals taken from the survey years 2004-2008 (48,921 observations) that contains 467 cases of death.

Figure 6: Predicted conditional survival probabilities by health status and education

Figure 6 shows the predicted conditional survival probabilities. When computing the individuals value functions, I need information on health transitions by education
up to age 99 that are not estimated by the health transition equation in the model. These transitions are computed non-parametrically in a first step using the SOEP data. This induces a minor inconsistency in the model in the sense that the health transition equation is a second order autoregressive process accounting also for heterogeneity by unobserved types. However, it is consistent with the assumption that individuals form their expectations about their mortality risk by health status and by education (high/low).
# Robustness checks

Table 10: Average effects of stochastic health shocks for alternative specifications

<table>
<thead>
<tr>
<th>Life-cycle outcome</th>
<th>Years of educ.&lt;12</th>
<th>Years of educ.≥12</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type 1</td>
<td>Type 2</td>
<td>Type 3</td>
</tr>
<tr>
<td></td>
<td>Type 1</td>
<td>Type 2</td>
<td>Type 3</td>
</tr>
<tr>
<td>Annuity market accounts for</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>differences in the longevity risk:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔRetirement age (years)</td>
<td>-5.8</td>
<td>-3.3</td>
<td>-1.4</td>
</tr>
<tr>
<td>ΔWork experience at ret. (years)</td>
<td>-5.2</td>
<td>-3.8</td>
<td>-3.8</td>
</tr>
<tr>
<td>ΔWealth at age 40 of non-retirees (€)</td>
<td>-4,014</td>
<td>-1,642</td>
<td>-1,320</td>
</tr>
<tr>
<td>ΔWealth at age 60 of non-retirees (€)</td>
<td>-17,103</td>
<td>-18,390</td>
<td>-20,640</td>
</tr>
<tr>
<td>Mean of average annual consumption (€)</td>
<td>20,867</td>
<td>15,555</td>
<td>10,760</td>
</tr>
<tr>
<td>ΔAverage annual consumption (%)</td>
<td>-5.1</td>
<td>-4.2</td>
<td>-4.8</td>
</tr>
<tr>
<td>Share in old-age poverty (%)</td>
<td>1.6</td>
<td>6.2</td>
<td>27.6</td>
</tr>
<tr>
<td>ΔShare in old-age poverty (pp)</td>
<td>+1.6</td>
<td>+5.2</td>
<td>+18.3</td>
</tr>
<tr>
<td>Alternative utility specification</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>and no offer and separation rates:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ΔRetirement age (years)</td>
<td>-0.1</td>
<td>-0.2</td>
<td>-3.0</td>
</tr>
<tr>
<td>ΔWork experience at ret. (years)</td>
<td>-0.2</td>
<td>-0.4</td>
<td>-5.0</td>
</tr>
<tr>
<td>ΔWealth at age 40 of non-retirees (€)</td>
<td>+1,947</td>
<td>+387</td>
<td>-1,578</td>
</tr>
<tr>
<td>ΔWealth at age 60 of non-retirees (€)</td>
<td>+9,937</td>
<td>+2,058</td>
<td>-9,379</td>
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<tr>
<td>Mean of average annual consumption (€)</td>
<td>23,685</td>
<td>17,101</td>
<td>9,886</td>
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<tr>
<td>ΔAverage annual consumption (%)</td>
<td>-2.8</td>
<td>-3.1</td>
<td>-10.6</td>
</tr>
<tr>
<td>Share in old-age poverty (%)</td>
<td>0.0</td>
<td>0.0</td>
<td>29.2</td>
</tr>
<tr>
<td>ΔShare in old-age poverty (pp)</td>
<td>+0.0</td>
<td>+0.0</td>
<td>+21.9</td>
</tr>
</tbody>
</table>

*Note:* The average effects are simulated on the basis of 5000 life-cycles. All individuals are assumed to enter the labor force in good health and non-employed. Hence, they only differ ex ante in terms of their unobserved type \( m \in \{1, 2, 3\} \) and educational attainment (draws are taken from the estimated distribution.)
References


*The Quarterly Journal of Economics*, 100(2), 389–408.


